

Higher moment singularities explored by the net proton non-statistical fluctuations

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We use the non-statistical fluctuation instead of the full one to explore the higher moment singularities of net proton event distributions in the relativistic Au+Au collisions at $\sqrt{s_{NN}}$ from 11.5 to 200 GeV calculated by the parton and hadron cascade model PACIAE. The PACIAE results of mean (M), variance (σ^2), skewness (S), and kurtosis (κ) are consistent with the corresponding STAR data. Non-statistical moments are calculated as the difference between the moments derived from real events and the ones from mixed events, which are constructed by combining particles randomly selected from different real events. An evidence of singularity at $\sqrt{s_{NN}} \sim 60$ GeV is first seen in the energy dependent non-statistical S and $S\sigma$.

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I. INTRODUCTION

Describing the QCD phase diagram as a function of temperature T and baryon chemical potential μ_B is one of the fundamental goals of heavy-ion collision experiments [1]. The finite temperature lattice QCD calculation at $\mu_B=0$ predicts that a crossover transition from the hadronic phase to Quark Gluon Plasma (QGP) phase may occur at temperature of 170-190 MeV [2, 3]. However, a QCD based model calculation indicates that the transition could be first order at large μ_B [4]. Once the location of the QCD critical point (QCP), where the phase transition proceeds from first order to crossover, is identified, the global structure of the phase diagram is known [5, 6].

A characteristic feature of QCP is the divergence of the correlation length ξ [7] and the extremely large critical fluctuations [8]. In a static and infinite medium, various moments of conserved quantities such as the net-baryon, net-charge, and net-strangeness are related to the correlation length ξ [9]. Typically the variance (σ^2) of these distributions is related to ξ as $\sigma^2 \sim \xi^2$ [8]. It is pointed out in [10] that higher moments of conserved quantity event distributions, measuring deviations from a Gaussian, are sensitive to the QCP fluctuations.

In the first quotation of [11] the hadronic and quark-gluon coexisting phases in a small volume have been investigated with a simple effective model. Assuming this finite system is in a heat reservoir, the order parameter (energy density e) fluctuation near the phase transition is studied in a way similar to the Landau theory. A positive skewness (S) was predicted and a negative kurtosis (κ)

was expected for the extensive thermodynamical quantities. In the second quotation of [11] the dynamical change of skewness and kurtosis was analyzed during hadronization of QGP. It was shown that the skewness changes from negative to positive during the transition. While the kurtosis is positive in the initial dominantly QGP phase and after hadronization, but it is negative during the process of transition. Similarly, it was predicted in the effective theory [10, 12] that a crossing of the phase boundary may result in a change of sign of skewness as a function of the energy density. It was also reported recently that the sign of kurtosis could be negative as well, if the QCP is approached from the crossover side of the QCD phase transition [13].

The products of higher moments, such as $S\sigma$ and $\kappa\sigma^2$, are related to the ratio of conserved quantity number susceptibilities (χ): $S\sigma \sim \chi^{(3)}/\chi^{(2)}$ and $\kappa\sigma^2 \sim \chi^{(4)}/\chi^{(2)}$ [14]. It is predicted that the conserved quantity event distribution becomes non-Gaussian and the susceptibility diverges when QCP is approached. This causes $S\sigma$ and $\kappa\sigma^2$ to change significantly.

Recently, QCP and the higher moments of conserved quantity event distribution in heavy ion collisions at BNL Relativistic Heavy Ion Collider (RHIC) energies have aroused further interest both experimentally [7, 15, 16] and theoretically [17–24]. On the experimental side, a method to determine T_c based on data has been proposed in [15]. By comparing the lattice results with the RHIC BES (Beam Energy Scan) fluctuation data of variance, skewness, and kurtosis systematically, the critical temperature is determined to be $T_c = 175_{-7}^{+1}$ MeV. However, no evidence of singularity in energy dependence (at given centrality) and/or centrality dependence (at given

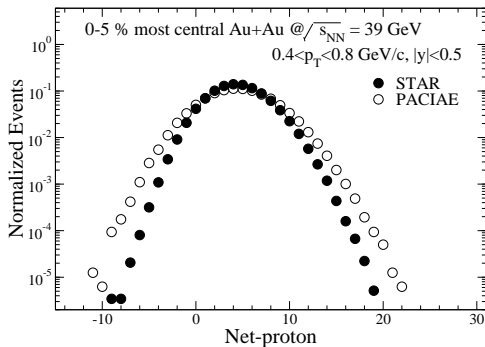


FIG. 1: Net-proton event distributions in Au+Au collisions at $\sqrt{s_{NN}}=39$ GeV. The solid and open symbols are STAR data (taken from first quotation of Ref. [16]) and the results of PACIAE real events, respectively.

energy) has been reported. On the other hand, copious models have been further proposed: such as the Lattice QCD [17], 2+1 flavor Quark-Meson model and Polyakov-Quark-Meson model [18], Polyakov-Nambu-Jona-Lasinio (PNJL) model [19, 20], Dyson-Schwinger equation [21], statistical model [22, 23], and the UrQMD and AMPT transport models [16, 24] etc. Each model has its merits and results, but the consistency is lacking and the contradiction is existing among the theoretical models. The difficulty is that the fluid dynamical development of energy, momentum, and baryon charge density leads to a complex spatial distribution of the measured and statistically analyzed quantities at the freeze-out and hadronization domain of the space-time. These also influence the measured higher moments even without a phase transition, and to disentangle the two effects is not easy [25]. Thus the question is still open and further studies are required.

In this paper a new method is studied to provide more insight. Namely, non-statistical moments of net proton event distribution are calculated as the difference between the moments derived from real events generated by the parton and hadron cascade model PACIAE [26] and the ones from mixed events which are randomly constructed according to the real events [27]. In this way the single particle distribution arising from the fluid dynamic development is separated from the two and more particle correlations stemming from the PACIAE model where the hadronization and freeze-out takes place, according to our expectation.

II. MODELS

The PACIAE model [26] is a parton and hadron cascade model, which is based on PYTHIA [28]. PACIAE consists of the parton initiation, parton evolution (rescattering), hadronization, and the hadron evolution (rescattering) four stages.

In the parton initiation stage, a nucleus-nucleus collision is decomposed into binary nucleon-nucleon (NN)

collisions according to the collision geometry and total NN cross section. The collision time is calculated for each NN collision pair assuming straight line trajectory between two consecutive NN collisions and the NN collision list is then constructed by all collision pairs. A NN collision with earliest collision time is selected from the collision list and performed by PYTHIA with string fragmentation switches-off and diquarks (anti-diquarks) break into quark pairs (anti-quark pairs). Thus a parton initial state (quarks, antiquarks, and gluons) is eventually obtained when NN collision pairs are exhausted.

The parton rescattering is then proceeded by Monte Carlo method using $2 \rightarrow 2$ leading order perturbative QCD cross sections [29]. The parton evolution stage is followed by the hadronization at the moment of partonic freeze-out (exhausting the partonic collisions). The Lund string fragmentation model and a phenomenological coalescence model are provided for hadronization. After this the rescattering among produced hadrons is dealt with the usual two body collision model [26]. Only the rescatterings among π , K , p , n , $\rho(\omega)$, Δ , Λ , Σ , Ξ , Ω , J/Ψ and their antiparticles are considered for simplicity.

Like other transport (cascade) models, such as above mentioned UrQMD [30] and/or AMPT [31], PACIAE does not assume equilibrium. It just simulates dynamically the whole relativistic heavy ion collision process from the initial partonic stage to the hadronic final state via the parton evolution (not implemented in UrQMD), hadronization, and hadron evolution according to a copious dynamical ingredients (assumptions) introduced reasonably. Therefore it is parallel to the experimental nucleus-nucleus collisions. These dynamics correctly describes the particle, energy, and entropy etc. developments, while intensive thermodynamical quantities are not defined in this non-equilibrium regime. Messages brought by the produced particles in these transport (cascade) models are all of dynamical origin. Unlike most of the hydrodynamic models where the phase transition is described via the assumptions for equation of states, in PACIAE (the same in UrQMD and/or AMPT) we do not implement a phase transition congenitally. However, once there is a phase transition signal in transport (cascade) model calculations, it must be a result of dynamical evolution. Of course, further studies are then required.

The n^{th} moment about the mean of a conserved quantity, x , with the event distribution $P(x)$ is expressed as

$$M^{(n)} = \langle (x - \langle x \rangle)^n \rangle = \int (x - \langle x \rangle)^n P(x) dx, \quad (1)$$

where the n^{th} moment about zero reads as

$$\langle x^{(n)} \rangle = \int x^n P(x) dx, \quad (2)$$

The higher moments as well as their products investigated widely are then

$$\text{variance :} \quad \sigma^2 = M^{(2)}, \quad (3)$$

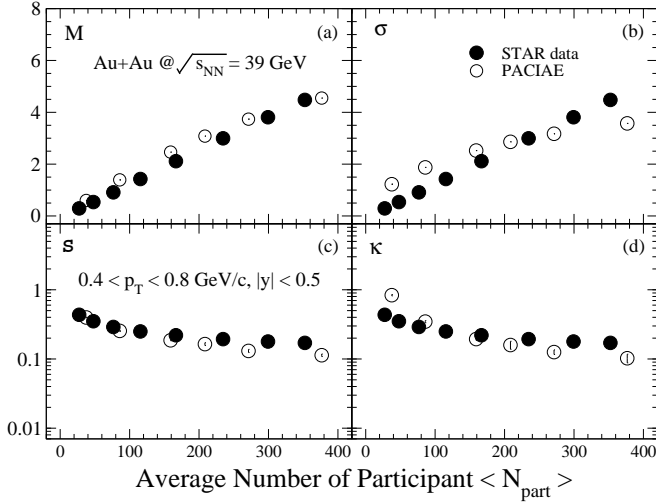


FIG. 2: Centrality dependent moments of net-proton event distribution in Au+Au collisions at $\sqrt{s_{NN}}=39$ GeV. The solid and open circles are STAR data (taken from first quotation of Ref. [16]) and the results of PACIAE real events, respectively.

$$\text{skewness : } S = M^{(3)}/(M^{(2)})^{3/2}, \quad (4)$$

$$\text{kurtosis : } \kappa = M^{(4)}/(M^{(2)})^2 - 3, \quad (5)$$

$$S\sigma = M^{(3)}/M^{(2)}, \quad (6)$$

and

$$\kappa\sigma^2 = M^{(4)}/M^{(2)} - 3M^{(2)}, \quad (7)$$

besides the mean $M \equiv \langle x^{(1)} \rangle$ and $M^{(1)} \equiv 0$.

The study of higher moment singularities is a matter of dynamics, but the statistical fluctuation is always dominant. Therefore, we study the non-statistical part of fluctuations instead of full fluctuations in this paper. The parton and hadron cascade model PACIAE [26] is applied to generate real events. Each real event obeys dynamical conservation laws (such as net baryon number conservation, energy and momentum conservations, etc.), which cause correlations among particles in a single event. As the PACIAE model simulation for a nucleus-nucleus collision is parallel to the experiment of a nucleus-nucleus collision, the NA49 method [27] is also employed to generate the mixed events. This means that the mixed events are constructed by combining particles randomly selected from different real events, while reproducing the event multiplicity distribution of real events. We have checked that the dynamical conservation laws really do not exist in the mixed event and there are only statistical fluctuations caused by the effects of finite event number, finite size, and the experimental finite detector resolutions. The “non-statistical moments” of conserved quantity event distributions are defined to be the difference between the moments derived from real events and the

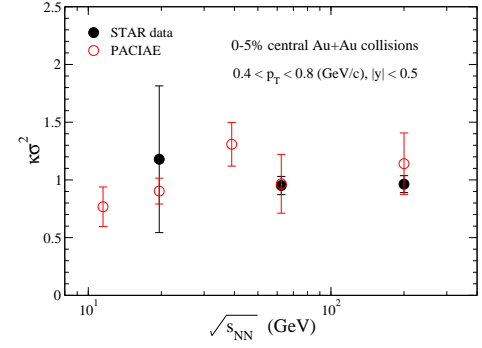


FIG. 3: (Color online) Energy dependence of $\kappa\sigma^2$ of net-proton event distribution in Au+Au collisions. The solid and open circles are STAR data (taken from [7]) and the results of PACIAE real events, respectively.

ones from mixed events. Thus the dynamical fluctuations are pronounced from the underlying dominant statistical fluctuations and are expected to be seen easily in the non-statistical higher moments.

If the n^{th} moment about the mean calculated by real (mixed) events is denoted by $M_R^{(n)}$ ($M_M^{(n)}$), then the corresponding n^{th} non-statistical moment about the mean, like in [27], is:

$$M_{NON}^{(n)} = M_R^{(n)} - M_M^{(n)}. \quad (8)$$

III. RESULTS

The net-proton event distribution and the corresponding higher moments in relativistic Au+Au collisions at $\sqrt{s_{NN}}$ from 11.5 to 200 GeV are calculated with both the PACIAE real events and the corresponding mixed events. It is found that the PACIAE results of mean, variance, skewness, and kurtosis in Au+Au collisions are in agreement with STAR data.

Shown, as an example, in Fig. 1 are the results of PACIAE real events (open symbols) compared with the STAR data (solid symbols) of net-proton event distribution in 0-5% most central Au+Au collision at $\sqrt{s_{NN}}=39$ GeV. Note that the spectator protons have been excluded here. One sees from Fig. 1 that the agreement between the STAR data and PACIAE results is satisfactory: with two distributions having nearly the same peak location and close each other until the half height. The deviation of the two distributions gets visible around the tails, however, the contribution of the tails to the moments is expected very small as the corresponding probability is rather low. In our knowledge there was not yet comparison between the STAR data of net-proton event distribution and model calculations, except the parameter fit in [22].

The centrality dependent STAR data of the moments of net-proton event distribution in Au+Au collision at

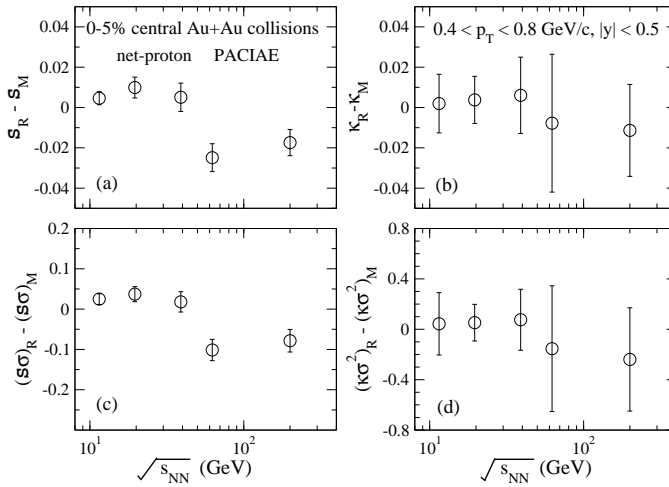


FIG. 4: Energy dependence of non-statistical higher moments and their products of the net-proton event distribution in 0-5% most central Au+Au collisions.

$\sqrt{s_{NN}}=39$ GeV (cf. first quotation of Ref. [16]) are compared with the results of PACIAE real events in Fig. 2. In this figure one sees the agreement between the STAR data and PACIAE results for M and S . However the agreement in σ and κ is not as good as in M and S . This reveals that the tails of the net-proton event distribution affect the even moments stronger.

Fig. 3 shows the energy dependence of the STAR $\kappa\sigma^2$ data and the results of PACIAE real events in Au+Au collisions. The STAR data characteristic feature of almost energy independent $\kappa\sigma^2$ is approximately reproduced by PACIAE within error bars.

We give in Fig. 4 the PACIAE results of energy dependence of the non-statistical skewness (panel a), kurtosis (b), $S\sigma$ (c), and $\kappa\sigma^2$ (d) calculated for the 0-5% most central Au+Au collisions in the STAR acceptances of $|y| < 0.5$ and $0.4 < p_T < 0.8$ GeV/c according to Eqs. (1-8). The energies applied in the calculations are $\sqrt{s_{NN}}=11.5, 19.6, 39, 62.4$ and 200 GeV, with the total number of generated real events being $3.0 \times 10^5, 3.0 \times 10^5, 2.4 \times 10^5, 1.2 \times 10^5$ and 1.2×10^5 , respectively. The same number of mixed events are then constructed correspondingly. It is found in this figure that the non-statistical S , κ , $S\sigma$, and $\kappa\sigma^2$ change sign at $\sqrt{s_{NN}} \sim 60$ GeV. Here a first evidence of singularity is seen in the energy dependent S and $S\sigma$. These signs, of course, are not seen in the results calculated by mixed events. They are also not showing up in the results calculated by real events and in the STAR data (results of full fluctuations indeed) [15, 32], because dynamical fluctuations are always submerged in the statistical fluctuations. It seems that the higher moment singularity be implicated in the non-statistical fluctuation.

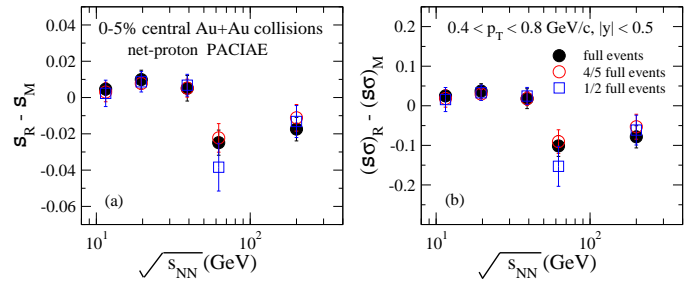


FIG. 5: (Color online) Energy dependence of non-statistical S and $S\sigma$ of the net-proton event distribution in 0-5% most central Au+Au collisions.

To check the reliability of the sign changes in Fig. 4, we have recalculated the energy dependent non-statistical S and $S\sigma$ with 1/2 and 4/5 total number of events and compared the results with the ones calculated with the full number of events, as shown in Fig 5. We see in this figure that the convergence is quite good for $\sqrt{s_{NN}}=11.5, 19.6, 39$ and 200 GeV and satisfactory for 62.4 GeV. Thus the results shown in Fig. 4 are reliable. The sign changes of S and $S\sigma$ shown in this figure may be a response to the prediction in [12].

The results in Fig. 4, when compared to the estimates in the second quotation of [11], indicate that the freeze-out point is very close to the hadronization point as the value of skewness is rather small. Furthermore, at lower beam energies the skewness is positive, indicating a freeze-out closer to the QGP side in the hadronization process. But at higher beam energies the skewness is negative, indicating more dominance of the hadronic side of the phase transition. The kurtosis becomes significantly negative, although small at higher beam energies, indicating that the freeze-out is indeed in the phase transition domain. Since a number of dynamical ingredients are introduced in the PACIAE (PYTHIA) model, the detailed roles and effects of the dynamical ingredients relevant to this phenomenon have to be studied later.

IV. CONCLUSION

In summary, we have calculated the higher moments of net-proton event distributions in relativistic Au+Au collisions at $\sqrt{s_{NN}}$ from 11.5 to 200 GeV with real events generated by the parton and hadron cascade model PACIAE [26]. The PACIAE results of centrality dependent net-proton M , σ , S , and κ in Au+Au collisions are consistent with the STAR data.

We have studied the non-statistical fluctuations of net-proton event distributions. The mixed events are constructed according to PACIAE real events and the non-statistical moments are calculated as the difference between the moment calculated from real events and the one from mixed events. The non-statistical S , κ , $S\sigma$, and $\kappa\sigma^2$ appear to change signs at $\sqrt{s_{NN}} \sim 60$ GeV. The

clear sign change in the energy dependent non-statistical S and $S\sigma$ may reflect some singularities. However, this has to be confirmed by the STAR Beam Energy Scan non-statistical fluctuation data later.

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